

In this article, we examine a means of implementing a well-known method of image processing, consisting in the following [1].

Let T (the field of view) be a set of number pairs (i, j) , where $i = \overline{1, n}$ and $j = \overline{1, m}$; the pair (i, j) will be called the cell with coordinates i and j .

The image V is a function that determines the value of the brightness $v(i, j)$ in each cell (i, j) of the field of view T .

Let configuration C be some subset of the field of view. In this case, the C -transformation of image V will be called the image X , the values of which are calculated using the rules

$$x(i, j) = \begin{cases} 1, & \text{if } \sum_{(k,l) \in C} v(i+k, j+l) \geq \theta; \\ 0, & \text{if } \sum_{(k,l) \in C} v(i+k, j+l) < \theta; \end{cases} \quad (1)$$

where θ is some threshold value, and the value of $x(i, j)$ is determined only in the case where for any pair $(k, l) \in C$ we have $(i+k, j+l) \in T$.

Transformations of this class are a repeatedly verified – by us as well – means of preliminary image processing. In particular, intelligent selection of configuration C and threshold θ permits image enhancement by eliminating, for example, defects in the form of small errors.

The C -transformation is often related to hopes for obtaining more significant results, up to detecting objects in the field of view that have a particular configuration in the sense of making a final decision and/or determining the coordinates of some symbol on a graphic image.

As a rule, these hopes are not realized in practice for the following reasons.

1. The image of any symbol unfortunately exhibits a very changeable configuration, and therefore, its reliable detection requires not one, but several fixed C -transformations.
2. Software for image transformation requires significant time even for a fixed configuration C , while hardware requires large quantities.

Thus, this image processing method may be applied to preliminary processing with the aim of enhancing the image and identifying places in the field of view that are "suspected" of having one or another configuration. To achieve such intermediate goals, the computational and hardware complexity is the fundamental obstacle in the path of applying the method under consideration [1].

Let us describe a set of means and methods that would allow us to obtain effective software for this class of transformations, which would remove the basic obstacles to their application.

Premises for Algorithm Design

In a system of discrete coordinates XOY , let us define a plane figure bounded by segments consisting of vertical and horizontal lines (Fig. 1) where each cell (i, j) having a brightness $v(i, j)$ is related to a unit square bounded by the lines $x = i + \Delta x/2$, $x = i - \Delta x/2$, $y = j + \Delta y/2$, $y = j - \Delta y/2$, where Δx , Δy are incremental steps of the abscissa and ordinate, respectively.

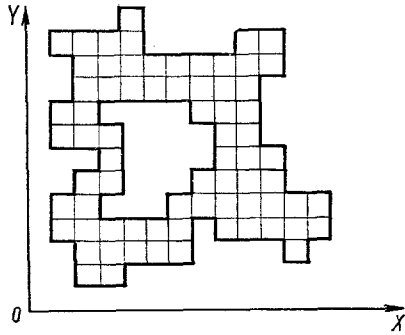


Fig. 1

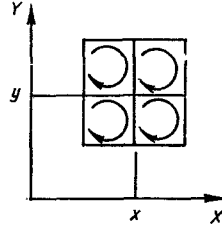


Fig. 2

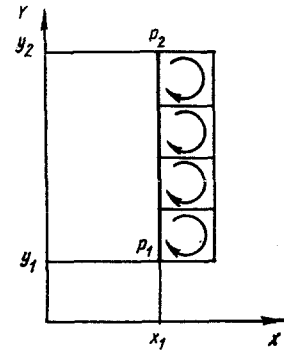


Fig. 3

We denote the set of pairs of discrete coordinates of the form (x, y) by \tilde{T} .

We assume that the given figure corresponds to configuration C , and pose the problem of calculating the overall brightness S_C for all cells of the given figure:

$$S_C = \sum_{(k,l) \in C} v(k, l). \quad (2)$$

With this aim, we define a so-called secondary image F in the points of set \tilde{T} , which represents a set of values $f(x, y)$, or F -values, that are specified at each point $(x, y) \in \tilde{T}$ as follows:

$$f(x, y) = \sum_{x \leftarrow x} \sum_{y \leftarrow y} v(x, y). \quad (3)$$

The following assertion is the basis of the proposed algorithm: to find the overall brightness S_C it is sufficient to determine the F -values at points where a vertical segment of the figure contour changes into a horizontal segment, or vice versa, and then to sum these values, each taken with its sign.

Let us prove the validity of this assertion. We transform the right-hand side of (3):

$$\begin{aligned} \sum_{x \leftarrow x} \sum_{y \leftarrow y} v(x, y) &= \sum_{x \leftarrow x - \Delta x} \sum_{y \leftarrow y - \Delta y} v(x, y) + \left[\sum_{x \leftarrow x - \Delta x} \sum_{y \leftarrow y} v(x, y) - \sum_{x \leftarrow x - \Delta x} \sum_{y \leftarrow y - \Delta y} v(x, y) \right] + \\ &+ \left[\sum_{x \leftarrow x} \sum_{y \leftarrow y - \Delta y} v(x, y) - \sum_{x \leftarrow x - \Delta x} \sum_{y \leftarrow y - \Delta y} v(x, y) \right] + v\left(x - \frac{\Delta x}{2}, y - \frac{\Delta y}{2}\right) = \sum_{x \leftarrow x - \Delta x} \sum_{y \leftarrow y} v(x, y) + \\ &+ \sum_{x \leftarrow x} \sum_{y \leftarrow y - \Delta y} v(x, y) - \sum_{x \leftarrow x - \Delta x} \sum_{y \leftarrow y - \Delta y} v(x, y) + v\left(x - \frac{\Delta x}{2}, y - \frac{\Delta y}{2}\right). \end{aligned}$$

Hereinafter, we set $\Delta x = \Delta y = 1$, and for convenience also consider the average cell brightness to be related not to the center, but to the upper right-hand corner, i.e., $v(x - \Delta x/2, y - \Delta y/2)$ is now denoted $v(x, y)$.

Thus, with the help of transformation (3) we arrive at a recursive formula for calculating the value $f(x, y)$:

$$f(x, y) = f(x - 1, y) + f(x, y - 1) - f(x - 1, y - 1) + v(x, y), \quad (4)$$

which allows the brightness inside a cell to be expressed as:

$$v(x, y) = f(x, y) - f(x, y - 1) + f(x - 1, y - 1) - f(x - 1, y), \quad (5)$$

i.e., to calculate the brightness inside one cell we must perform a clockwise trace of its contour and then, from the sum of its F -values, which are determined from the starting points (i.e., the points first encountered) of the vertical contour segments, to calculate the sum of the values found for starting points of horizontal segments.

Since the method of determining brightness inside one cell is known, we will use this method to calculate the values of the overall brightness S_C for all cells of the given figure. Here, each value $f(x, y)$, defined at a node of a coordinate network within the figure contour, may contribute to the sum for determining the quantities S_C from one to four times with

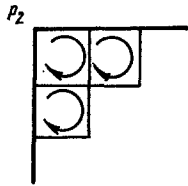


Fig. 4

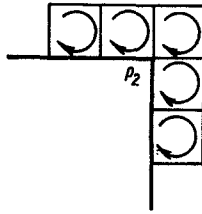


Fig. 5

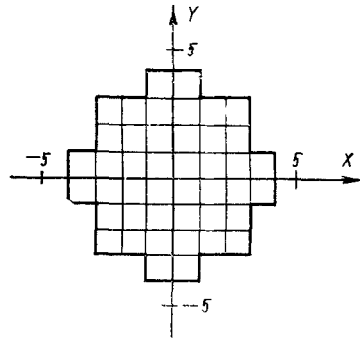


Fig. 6

different signs in accordance with the number of cells in the figure using this node as an apex for one of its angles.

Each value of $f(x, y)$ for a point lying inside and not on the figure contour corresponds to a node in the coordinate network, used as an apex by four pairwise adjacent cells (Fig. 2); a result of the same direction of traversal for each of the cells is that $f(x, y)$ twice enters positively and twice negatively into the sum for the determination of S_C .

Thus, the finite result of combining F -values for points lying inside the figure contour will be zero.

This is also valid for values of $f(x, y)$ at points lying on the figure contour and used as apices of two adjacent cells.

Consequently, in determining the overall brightness S_C , only those values of $f(x, y)$ that correspond to points where the vertical segments become horizontal (or vice versa) are significant. Such points are hereinafter called defining.

We now go over to the question of the sign of values $f(x, y)$ at defining points. We observe an arbitrary vertical segment of the figure contour, bounded by points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, and let the figure cells lie to the right of the segment under discussion (Fig. 3).

Regardless of the orientation of adjacent horizontal segments the value $f(x_1, y_1)$ enters into the sum with a plus sign, while $f(x_1, y_2)$ enters with a minus sign.

We show this using point P_2 as an example. Actually, let the horizontal segment of the contour be directed to the right from this point (Fig. 4). In this case, only one cell uses the point P_2 as an apex, and in accordance with the accepted direction of cell transversal, $f(x_1, y_2)$ will be negative.

If the horizontal segment from point P_2 is directed to the left (Fig. 5), then the appearance of another two adjacent cells that use point P_2 as an apex of one of their corners does not lead to any changes since one of these cells $f(x_1, y_2)$ is negative; the other, positive.

Performing similar analysis for the case where figure cells lie to the left of the vertical segment, and/or having examined horizontal segments the same way, it is easy to see that if we number defining points in the order of contour traversal, then points with even numbers will correspond to values of $f(x, y)$ with signs different from those of such values at points with odd numbers within the limits of the same contour.

Thus, the sought-after value for the overall brightness S_C may be expressed using the values $f(x, y)$ of the form in (2) in the following form:

$$S_C = \sum_{i=1}^N C_i f(x_i, y_i). \quad (6)$$

Here N is the number of defining points over the entire figure contour, x_i, y_i are the coordinates of the i -th defining point, and C_i is equal to either $+1$ or -1 , depending on the number i of the defining point and the accepted numbering method.

The following rule may, for example, be adopted: the outside figure contours are traversed in a clockwise direction; internal contours, in a counterclockwise direction. At the same time four numbers must correspond to the starting points of the vertical segments; then $C_i = (-1)^i$.

TABLE 1

Point number, i	Abscissa, x_i	Ordinate, y_i	Sign multiplier, C_i
1	-1	-3	-1
2	-3	-3	+1
3	-3	-1	-1
4	-4	-1	+1
5	-4	1	-1
6	-3	1	+1
7	-3	3	-1
8	-1	3	+1
9	-1	4	-1
10	1	4	+1
11	1	3	-1
12	3	3	+1
13	3	1	-1
14	4	1	+1
15	4	-1	-1
16	3	-1	+1
17	3	-3	-1
18	1	-3	+1
19	1	-4	-1
20	-1	-4	+1

Now, relying on the result in (6), we describe in detail the technique of the C-transformation for a processed image in accordance with (1).

Description of the Algorithm

We will accompany the algorithm description with an example of solving the formulated problem of identifying places on the image that are suspected of having a specified symbol.

As an example, we take the case when the analyzed image is two-level, the configuration C has no internal contours, and finally, threshold θ [see (1)] is chosen equal to the number of cells in configuration C. Solution $x(i, j) = 1$, i.e., the solution that the sought-after configuration has been detected on the analyzed image, obtains only when all cells with coordinates $(i + k, j + l)$, $(k, l) \in C$, are blackened.

The above conditions are actually valid, for example, in the practical case of detecting contact fields on printing plate phototemplates, where the configuration of the contact field, i.e., the specimen symbol, either has no internal contours, or may be ignored owing to unblackened cells in the specimen are sometimes blackened.

TABLE 2

	18	0	0	0	3	11	18	27	37	45	52	57	62	68	73	78	83	93	103	113	122	131	136	146	150
	17	0	0	0	3	11	18	27	37	45	52	57	62	68	73	78	83	93	103	112	120	128	135	143	147
	16	0	0	0	3	11	18	27	37	45	52	57	62	68	73	78	83	93	102	110	117	124	130	137	141
	15	0	0	0	3	11	18	27	37	45	52	57	62	68	73	78	83	93	101	108	114	120	125	131	134
	14	0	0	0	3	11	18	27	37	45	52	57	62	68	73	78	83	92	99	105	110	115	119	124	126
	13	0	0	0	3	11	18	27	37	45	52	57	62	68	73	78	83	91	97	102	106	110	113	117	118
	12	0	0	0	3	11	18	27	37	45	52	57	62	68	73	78	83	90	95	99	102	105	107	110	110
	11	0	0	0	3	11	18	27	37	45	52	57	62	68	73	78	83	89	93	96	98	100	101	103	103
	10	0	0	0	3	11	18	27	36	44	51	56	61	67	72	76	80	85	88	90	91	92	92	93	93
	9	0	0	0	3	11	18	26	34	41	48	53	58	63	67	70	73	77	79	80	80	80	80	80	80
	8	0	0	0	3	10	16	23	30	36	42	47	51	55	58	60	62	65	66	66	66	66	66	66	66
	7	0	0	0	3	9	14	20	26	31	36	40	43	46	48	49	50	52	52	52	52	52	52	52	52
	6	0	0	0	2	7	11	16	21	25	29	32	34	36	37	37	37	38	38	38	38	38	38	38	38
	5	0	0	0	1	5	8	12	16	19	22	24	25	25	26	26	26	26	26	26	26	26	26	26	26
	4	0	0	0	0	3	5	8	11	13	15	16	16	16	16	16	16	16	16	16	16	16	16	16	16
	3	0	0	0	0	2	3	5	7	8	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
	2	0	0	0	0	1	1	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23

This means that for all points of the image with coordinates (\tilde{x}, \tilde{y}) we must calculate the quantity

$$S_C(\tilde{x}, \tilde{y}) = \sum_{i=1}^N f(\tilde{x} + x_i, \tilde{y} + y_i) C_i, \quad (7)$$

and if one of the coordinates from the pair $(\tilde{x} + x_i, \tilde{y} + y_i)$ falls outside the field of view, then we will consider the quantity $S_C(\tilde{x}, \tilde{y})$ to be undefined.

For this case $\tilde{x} = 0, 1, 2, \dots, 23$, $\tilde{y} = 0, 1, 2, \dots, 18$, the number of defining points $N = 20$, and their coordinates x_i, y_i and sign multipliers C_i are given in Table 2.

The results of calculations performed in accordance with (7) are presented in Table 3.

In the final step, we equate the values found in accordance with (7) with the specimen values S_{sp} , and where they are equal

$$S_C(\tilde{x}, \tilde{y}) = S_{sp} \quad (8)$$

we accept the hypothesis that a specified specimen symbol may be found at the image point having coordinates (\tilde{x}, \tilde{y}) , i.e., if the specimen base point is made coincident with (\tilde{x}, \tilde{y}) for parallel and identically directed axes of both the specimen and image, then each blackened specimen cell coincides with a blackened cell in the image. In the opposite case, the hypothesis is rejected.

In this case, the role of the threshold value θ is played by S_{sp} , and the points of the initial image, for which (8) is valid, determine [in accordance with (1)] the image X , obtained by a C-transformation of the initial image V via the method described above.

For the given example, such a transformed image is shown in Fig. 8.

Let us estimate the speed of the proposed algorithm. In accordance with (4), calculation of $f(x, y)$ for each image point requires four operations; consequently, for an $m \times n$ image, step 3 will require $4mn$ operations.

Calculation of $S_C(\tilde{x}, \tilde{y})$ in step 4 in accordance with (7) for each image point, where this quantity is defined, requires N operations.

Consequently, the order of the number of operations to execute the algorithm may be estimated as $O(Nmn)$, where N is the number of defining points in the specimen symbol, and $m \times n$ is the number of points in the analyzed image.

Thus, the speed of the proposed algorithm, by comparison with known methods [1], is as many times higher as the number of defining points of the symbol is less than the overall number of its blackened cells, and the solution of the problem for an arbitrary number of specimen symbols for the same image requires only the execution of step 3 of the given algorithm.

In conclusion, we note that selection of the configuration of the specimen symbol and the threshold values must be experimentally supported so as to eliminate the influence of unavoidable distribution error and noise in real images. In particular, the specimen configuration, shown in Fig. 5, was constructed from a discretized image of a contact field, i.e., a blackened circle with radius 5. However, in more complex cases that may occur in practice, selection of configuration and threshold value significantly depends on the makeup of the entire collection of specimens and requires particular examination.

LITERATURE CITED

1. R. O. Duda and P. E. Hart, Pattern Classification and Scheme Analysis, Wiley-Interscience (1973).